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## MASSSES OF CONSTITUENT QUARKS CONFINED IN OPEN BOTTOM HADRONS

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We apply color-spin and flavor-spin quark-quark interactions to the meson and baryon constituent quarks, and calculate constituent quark masses, as well as the coupling constants of these interactions. The main goal of this paper was to determine constituent quark masses from light and open bottom hadron masses, using the fitting method we have developed and clustering of hadron groups. We use color-spin Fermi-Breit (FB) and flavor-spin Glazman-Riska (GR) hyperfine interaction (HFI) to determine constituent quark masses (especially  $b$  quark mass). Another aim was to discern between the FB and GR HFI because our previous findings had indicated that both interactions were satisfactory. Our improved fitting procedure of constituent quark masses showed that on average color-spin (Fermi-Breit) hyperfine interaction yields better fits. The method also shows the way how the constituent quark masses and the strength of the interaction constants appear in different hadron environments.

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## 1. Introduction

Determination of quark masses is extremely important, for both phenomenological and theoretical applications.<sup>1</sup> About the importance of the mass of the bottom quark, as a fundamental parameter of the Standard Model, see also review by El-Khadra and Luke.<sup>2</sup>

Many spectroscopic quark models, based on two-body interactions, have been developed (see e.g. Ref. 3 and references therein). These are called “hyperfine interaction” (HFI) in analogy with the atomic physics. The simplest HFI is the spin-spin interaction proposed by Rujula, Georgi and Glashow back in 1975,<sup>4</sup> which was a major advance beyond the naive quark model and it is presented in many textbooks. The spin-color or Fermi-Breit (FB) model,<sup>5</sup> although it leads to the same mass formulas for conventional hadrons and it is still phenomenological, incorporates the hypothesis of color. In paper of Glozman and Riska<sup>6</sup> it was shown that spin-flavor HFI is suitable for light quarks. They analyze a baryon spectrum in terms of an  $SU(3)$  flavor-symmetric quark-quark interaction that describes chiral pseudoscalar boson exchange. In this paper, the hyperfine interactions (HFIs) between constituent quarks in mesons and baryons, including those with one  $b$ -quark, are used to investigate how their masses are affected by these interactions and to compare theoretically obtained masses with experimentally measured ones. The two types of interactions: Fermi-Breit (FB)<sup>4,5,7,8,9,10,11,12</sup> and Glozman-Riska (GR)<sup>13,6,14,10,11,15</sup> (i.e. color-spin and flavor-spin) are used to obtain meson and baryon mass formulas. Then we compare our theoretical calculations with known masses of  $b$ -hadrons.

Detailed physical justification of the models is beyond the scope of this paper. For example, the models are non-relativistic and they neglect the kinetic energy. Nevertheless, they have several advantages, e.g. they are rather explicit since they yield elementary expressions for the hadron masses, they have few free parameters etc. Any more realistic model necessarily introduces additional parameters; even the original Rujula-Georgi-Glashow model contained additional terms with respect to the FB and GR models used here (see Ref. 4 or formula (7.75) in Ref. 16). Moreover, bound states in a realistic two-body potential can only be found numerically so the dependence of theoretical hadron masses on the parameters of the model is not as clear as in the simple models. A model with two additional free parameters – given that few (statistical) degrees of freedom would be left – would have to give a very good fit to the observed hadron masses in order to be considered satisfactory.

Although FB and GR HFI are well known,<sup>4,13</sup> we use these interactions for determining the constituent quark masses (especially  $b$  quark mass) since we were able to find few papers where constituent  $b$  quark mass is determined.<sup>17,18,19,20,21,22,23</sup> Also, our approach differs from the ones used in those papers. Moreover, the ‘constituent’ quark mass as used here is not exactly the same as the ‘bottom  $1S$ ’ mass often defined (using bottomonium) in such determinations, e.g. in Ref. 24. We also provide explicit mass formulas for hadrons containing  $b$  quark using both HFIs.

In this paper we make one of the first attempts to estimate uncertainties of the constituent quark masses. This estimation was partly motivated by the need to discern between the physically distinct FB and GR HFI because both had yielded satisfactory fits of the hadron masses in some cases.<sup>11</sup>

One of our goals is to investigate the models that are simple enough to analyze the exotic hadron states – primarily the tetraquarks – because more elaborate models, having far less transparent dependence on model parameters, become quite difficult to analyze (if not to compute) in systems with several quark pairs.

This paper is organized as follows: in Sec. 2 we present the two strong hyperfine interactions, then in Sec. 3 we give their influence on hadron masses and derive formulas; the method of our calculation of the constituent quark masses and the coupling constants (least-square fit) is given in Sec. 4, in Sec. 5 the different combinations of hadron mass equations are solved (clustering of hadron groups), and in Sec. 6 we discuss the obtained results. We point out the main conclusions in Sec. 7.

## 2. Strong hyperfine interactions and the schematic model

The main interaction which binds quarks into groups (hadrons) depends on the color and the spin. With no hyperfine interaction added, there would be degenerate hadrons with different spins. To avoid this spin degeneration, hyperfine interaction is included and it depends, among other properties, on the spin too.

Strong Fermi-Breit hyperfine interaction Hamiltonian<sup>5</sup> with SU(3) flavor symmetry breaking is of the form:

$$H_{\text{FB}} = C \sum_{i < j} \left( \frac{\vec{\sigma}_i \vec{\sigma}_j}{m_i m_j} \right) (\lambda_i^C \lambda_j^C), \quad (1)$$

where  $m_i$  are constituent masses of the interacting quarks,  $\sigma_i$  are the Pauli spin matrices,  $\lambda_i^C$  are the color Gell-Mann matrices and  $C$  is a constant. This interaction is also called color-spin interaction. As explained in the papers<sup>4,25</sup>, Fermi-Breit interaction originates from one gluon exchange between two bodies, in analogy with the photon exchange between charged Dirac particles. The Fermi term of this interaction refers to hyperfine splitting of masses, i.e. it depends on inverse product of the quark masses, while Breit interaction contains a part which is spin-dependent (short range gluon interactions) and another spin-independent part (forces that keep  $q\bar{q}$  pairs in color singlets). In the case of this interaction, we neglect all other potentials in the system, and include only Fermi-Breit two particle interaction.

The formulas derived from (1) reduce to the simplified Rujula-Georgi-Glashow model as the expectation values of the products  $\lambda_i^C \lambda_j^C$  can be absorbed into the HFI constants. However, the models are not equivalent; not only does  $H_{\text{FB}}$  include the hypothesis of color explicitly, but it also leads to different mass formulas for exotic hadrons since the products  $\lambda_i^C \lambda_j^C$  are different for  $qq$  and  $q\bar{q}$  pairs.

Strong Glozman-Riska Hamiltonian<sup>6</sup> is of the form:

$$H_{\text{GR}} = -C_\chi \sum_{i < j} (-1)^{\alpha_{ij}} \left( \frac{\vec{\sigma}_i \vec{\sigma}_j}{m_i m_j} \right) (\lambda_i^F \lambda_j^F); \quad (-1)^{\alpha_{ij}} = \begin{cases} -1, q\bar{q} \\ +1, qq \text{ or } \bar{q}\bar{q} \end{cases}, \quad (2)$$

where  $\lambda_i^F$  are Gell-Mann matrices for flavor SU(3),  $\sigma_i$  are the Pauli spin matrices and  $C_\chi$  is a constant. This is flavor-spin interaction. This interaction between constituent quarks describes pseudoscalar boson exchange, i.e. fine structure of the spectrum is based on the interaction mediated by the SU(3)<sub>F</sub> octet of pseudoscalar mesons, which are the Goldstone bosons.

We employ these schematic color-spin and flavor-spin interactions between quarks and antiquarks which lead to hyperfine interaction contributions to the meson and baryon masses. The schematic approximation means that we used two-particle interaction: in our calculation of hadron masses, we pay attention only to the short-range forces which arise from one-gluon exchange, i.e. the hadron masses are described in terms of two-body quark-quark forces.

### 3. Hadron masses with FB and GR HFIs

The contribution of HFI to hadron masses would be  $m_{\nu, \text{HFI}} = \langle \nu | \langle \chi | H_{\text{HFI}} | \chi \rangle | \nu \rangle$ , where  $\chi$  denotes the spin wave function and  $\nu$  the flavor wave function while HFI is either FB or GR interaction. For total hadron masses  $m_\nu$  we have  $m_\nu = m_{\nu,0} + m_{\nu, \text{HFI}}$ , where  $m_{\nu,0}$  are masses without influence of HFI.

Experimentally detected hadrons are listed in the Summary Tables of the Particle Data Group (PDG).<sup>26</sup> Among them, we choose the particles with orbital momentum  $L = 0$ , and with a certain total momentum  $J = L + S$  ( $S$  being spin) and the parity  $P$  (note that  $P = (-1)^{L+1}$  for mesons and  $P = (-1)^L$  for baryons).

Among the mesons listed in Particle Physics Summary Tables, we choose the following particles:

- light pseudoscalar mesons  $J^P = 0^-$ ;  $S = 0$ :  
 $\pi^+$ ,  $\pi^0$ ,  $\pi^-$ ,  $K^+$ ,  $K^0$ ,  $\bar{K}^0$ ,  $K^-$  (note that we did not take into account  $\eta$  and  $\eta'$  because their mixing changes their properties as well as their masses),
- light vector mesons  $J^P = 1^-$ ;  $S = 1$ :  
 $\rho^+$ ,  $\rho^0$ ,  $\rho^-$ ,  $K^{*+}$ ,  $K^{*0}$ ,  $\bar{K}^{*0}$ ,  $K^{*-}$ ,  $\omega$ ,  $\phi$ ,
- bottom mesons  $J^P = 0^-$ ;  $J^P = 1^-$ :  
 $B^+$ ,  $B^0$ ,  $\bar{B}^0$ ,  $B^-$ ,  $B^*$ ,
- strange bottom mesons  $J^P = 0^-$ ;  $J^P = 1^-$ :  
 $B_S^0$ ,  $\bar{B}_S^0$ ,  $B_S^*$ .

We first present equations for theoretical meson masses with FB HFI included obtained from (1). We denote the constant for this interaction for mesons by  $C^m$ . The possible small mass difference between  $u$  and  $d$  constituent quarks is neglected. However, the difference in the observed masses within the isospin multiplets is taken

into account (except for  $\Delta$ -baryons, for which an average mass is used) so, in those cases, the same theoretical masses are fitted to different observed ones.

$$m_{\pi^\pm}^{\text{th}} = 2m_u - \frac{3C^m}{m_u^2} = m_{\pi^0}^{\text{th}}, \quad m_{K^\pm}^{\text{th}} = m_u + m_s - \frac{3C^m}{m_u m_s} = m_{K^0}^{\text{th}} = m_{\bar{K}^0}^{\text{th}}. \quad (3)$$

$$\begin{aligned} m_{\rho^\pm}^{\text{th}} &= m_{\rho^0}^{\text{th}} = 2m_u + \frac{C^m}{m_u^2} = m_\omega^{\text{th}}, & m_\phi^{\text{th}} &= 2m_s + \frac{C^m}{m_s^2}, \\ m_{K^{*+}}^{\text{th}} &= m_{K^{*-}}^{\text{th}} = m_u + m_s + \frac{C^m}{m_u m_s} = m_{K^{*0}}^{\text{th}} = m_{\bar{K}^{*0}}^{\text{th}}. \end{aligned} \quad (4)$$

$$m_{B^+}^{\text{th}} = m_{B^-}^{\text{th}} = m_u + m_b - \frac{3C^m}{m_u m_b} = m_{B^0}^{\text{th}} = m_{\bar{B}^0}^{\text{th}}, \quad m_{B^*}^{\text{th}} = m_u + m_b + \frac{C^m}{m_u m_b}. \quad (5)$$

$$m_{B_s^0}^{\text{th}} = m_s + m_b - \frac{3C^m}{m_s m_b} = m_{\bar{B}_s^0}^{\text{th}}, \quad m_{B_s^*}^{\text{th}} = m_s + m_b + \frac{C^m}{m_s m_b}. \quad (6)$$

Now we give masses with GR from (2) where the constant is denoted by  $C_\chi^m$ .

$$m_{\pi^\pm}^{\text{th}} = 2m_u - \frac{2C_\chi^m}{m_u^2} = m_{\pi^0}^{\text{th}}, \quad m_{K^\pm}^{\text{th}} = m_u + m_s - \frac{2C_\chi^m}{m_u m_s} = m_{K^0}^{\text{th}} = m_{\bar{K}^0}^{\text{th}}. \quad (7)$$

$$\begin{aligned} m_{\rho^\pm}^{\text{th}} &= m_{\rho^0}^{\text{th}} = 2m_u + \frac{2C_\chi^m}{3m_u^2} = m_\omega^{\text{th}}, & m_\phi^{\text{th}} &= 2m_s - \frac{16C_\chi^m}{3m_s^2}, \\ m_{K^{*+}}^{\text{th}} &= m_{K^{*-}}^{\text{th}} = m_u + m_s + \frac{2C_\chi^m}{3m_u m_s} = m_{K^{*0}}^{\text{th}} = m_{\bar{K}^{*0}}^{\text{th}}. \end{aligned} \quad (8)$$

$$m_{B^+}^{\text{th}} = m_{B^-}^{\text{th}} = m_u + m_b - \frac{2C_\chi^m}{m_u m_b} = m_{B^0}^{\text{th}} = m_{\bar{B}^0}^{\text{th}}, \quad m_{B^*}^{\text{th}} = m_u + m_b + \frac{2C_\chi^m}{3m_u m_b}. \quad (9)$$

$$m_{B_s^0}^{\text{th}} = m_s + m_b - \frac{2C_\chi^m}{m_s m_b} = m_{\bar{B}_s^0}^{\text{th}}, \quad m_{B_s^*}^{\text{th}} = m_s + m_b + \frac{2C_\chi^m}{3m_s m_b}. \quad (10)$$

From the baryons listed by the PDG,<sup>26</sup> we choose these particles (note that the subscripts indicate heavy quark content, and in our case subscript  $b$  indicates content of one  $b$  quark):

- light baryons - octet with mixed symmetry  $J^P = 1/2^+$ :  
 $p, n, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-, \Lambda$ ,
- light baryons - symmetric decuplet  $J^P = 3/2^+$ :  
 $\Delta^{++}, \Delta^+, \Delta^0, \Delta^-, \Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}, \Xi^{*0}, \Xi^{*-}, \Omega$ ,
- bottom baryons  $J^P = 1/2^+; J^P = 3/2^+$ :  
 $\Sigma_b^+, \Sigma_b^-, \Lambda_b, \Sigma_b^{*+}, \Sigma_b^{*-}, \Omega_b$ .

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We give their theoretical masses with FB HFI influence by Eqs. (11)-(13). Constant for the FB HFI for baryons is denoted by  $C^b$ .

$$\begin{aligned} m_p^{\text{th}} &= 3m_u - 3C^b \frac{1}{2m_u^2} = m_n^{\text{th}}, & m_\Lambda^{\text{th}} &= 2m_u + m_s - 3C^b \frac{1}{2m_u^2}, \\ m_{\Sigma^+}^{\text{th}} &= 2m_u + m_s + 2C^b \frac{1}{m_u^2} \left( \frac{1}{4} - \frac{m_u}{m_s} \right) = m_{\Sigma^0}^{\text{th}} = m_{\Sigma^-}^{\text{th}}, \\ m_{\Xi^0}^{\text{th}} &= m_u + 2m_s + 2C^b \frac{1}{m_s^2} \left( \frac{1}{4} - \frac{m_s}{m_u} \right) = m_{\Xi^-}^{\text{th}}. \end{aligned} \quad (11)$$

$$\begin{aligned} m_{\Delta^{++}}^{\text{th}} &= m_{\Delta^+}^{\text{th}} = m_{\Delta^0}^{\text{th}} = m_{\Delta^-}^{\text{th}} = 3m_u + 3C^b \frac{1}{2m_u^2}, \\ m_{\Sigma^{*+}}^{\text{th}} &= 2m_u + m_s + C^b \frac{1}{m_u^2} \left( \frac{1}{2} + \frac{m_u}{m_s} \right) = m_{\Sigma^{*0}}^{\text{th}} = m_{\Sigma^{*-}}^{\text{th}}, \\ m_{\Xi^{*0}}^{\text{th}} &= m_u + 2m_s + C^b \frac{1}{m_s^2} \left( \frac{1}{2} + \frac{m_s}{m_u} \right) = m_{\Xi^{*-}}^{\text{th}}, \\ m_\Omega^{\text{th}} &= 3m_s + 3C^b \frac{1}{2m_s^2}. \end{aligned} \quad (12)$$

$$\begin{aligned} m_{\Sigma_b^+}^{\text{th}} &= 2m_u + m_b + 2C^b \frac{1}{m_u^2} \left( \frac{1}{4} - \frac{m_u}{m_b} \right) = m_{\Sigma_b^-}^{\text{th}}, \\ m_{\Lambda_b}^{\text{th}} &= 2m_u + m_b - 3C^b \frac{1}{2m_u^2}, \\ m_{\Sigma_b^{*+}}^{\text{th}} &= 2m_u + m_b + C^b \frac{1}{m_u^2} \left( \frac{1}{2} + \frac{m_u}{m_b} \right) = m_{\Sigma_b^{*-}}^{\text{th}}, \\ m_{\Omega_b}^{\text{th}} &= 2m_s + m_b + C^b \frac{1}{m_s^2} \left( \frac{1}{2} + \frac{m_s}{m_b} \right). \end{aligned} \quad (13)$$

Now we give masses with GR HFI. The GR HFI constant is denoted by  $C_\chi^b$ .

$$\begin{aligned} m_p^{\text{th}} &= 3m_u - 8C_\chi^b \frac{1}{m_u^2} = m_n^{\text{th}}, & m_\Lambda^{\text{th}} &= 2m_u + m_s - C_\chi^b \frac{1}{3m_u^2} \left( 13 + \frac{11m_u}{m_s} \right), \\ m_{\Sigma^+}^{\text{th}} &= 2m_u + m_s - C_\chi^b \frac{1}{m_u^2} \left( 1 + \frac{7m_u}{m_s} \right) = m_{\Sigma^0}^{\text{th}} = m_{\Sigma^-}^{\text{th}}, \\ m_{\Xi^0}^{\text{th}} &= m_u + 2m_s - C_\chi^b \frac{1}{m_s^2} \left( 1 + \frac{7m_s}{m_u} \right) = m_{\Xi^-}^{\text{th}}. \end{aligned} \quad (14)$$

$$\begin{aligned} m_{\Delta^{++}}^{\text{th}} &= m_{\Delta^+}^{\text{th}} = m_{\Delta^0}^{\text{th}} = m_{\Delta^-}^{\text{th}} = 3m_u - \frac{4C_\chi^b}{m_u^2}, \\ m_{\Sigma^{*+}}^{\text{th}} &= 2m_u + m_s - 8C_\chi^b \frac{1}{3m_u^2} \left( \frac{1}{2} + \frac{m_u}{m_s} \right) = m_{\Sigma^{*0}}^{\text{th}} = m_{\Sigma^{*-}}^{\text{th}}, \\ m_{\Xi^{*0}}^{\text{th}} &= m_u + 2m_s - 8C_\chi^b \frac{1}{3m_s^2} \left( \frac{1}{2} + \frac{m_s}{m_u} \right) = m_{\Xi^{*-}}^{\text{th}}, \\ m_\Omega^{\text{th}} &= 3m_s - 4C_\chi^b \frac{1}{m_s^2}. \end{aligned} \quad (15)$$

$$\begin{aligned}
m_{\Sigma_b^+}^{\text{th}} &= 2m_u + m_b - C_\chi^b \frac{1}{m_u^2} \left( 1 + \frac{7m_u}{m_b} \right) = m_{\Sigma_b^-}^{\text{th}}, \\
m_{\Lambda_b}^{\text{th}} &= 2m_u + m_b - C_\chi^b \frac{1}{3m_u^2} \left( 13 + \frac{11m_u}{m_b} \right), \\
m_{\Sigma_b^{*+}}^{\text{th}} &= 2m_u + m_b - 8C_\chi^b \frac{1}{3m_u^2} \left( \frac{1}{2} + \frac{m_u}{m_b} \right) = m_{\Sigma_b^{*-}}^{\text{th}}, \\
m_{\Omega_b}^{\text{th}} &= 2m_s + m_b - 8C_\chi^b \frac{1}{3m_s^2} \left( \frac{1}{2} + \frac{m_s}{m_b} \right).
\end{aligned} \tag{16}$$

Table 1. Fitted values of constituent quark masses  $m_u$  ( $= m_d$ ),  $m_s$ ,  $m_b$  (MeV) and the hyperfine constants  $C^m$  and  $C_\chi^m$  ( $10^7$  MeV<sup>3</sup>), obtained by  $\chi^2$  fits of meson masses with FB and GR HFI.

Fit No.	HFI	Mesons	Quark masses (MeV)			Constant ( $\times 10^7$ MeV <sup>3</sup> )
			$m_u = m_d$	$m_s$	$m_b$	
1	FB	$\pi, K, \rho, K^*, \omega, \phi,$	$307.54 \pm 1.16$	$487.41 \pm 1.56$	$4967.20 \pm 18.73$	$1.50 \pm 0.02$
	GR	$B, B^*, B_S, B_S^*$	$293.09 \pm 11.61$	$513.00 \pm 15.84$	$4964.87 \pm 189.22$	$1.93 \pm 0.26$
2	FB	$\pi, K, \rho, K^*, \omega, \phi$	$307.49 \pm 1.19$	$487.52 \pm 1.59$	-	$1.50 \pm 0.02$
	GR		$293.05 \pm 14.36$	$513.15 \pm 19.59$	-	$1.93 \pm 0.32$

Table 2. The same as Table 1, but for baryon fit.

Fit No.	HFI	Baryons	Quark masses (MeV)			Constant ( $\times 10^7$ MeV <sup>3</sup> )
			$m_u = m_d$	$m_s$	$m_b$	
1	FB	$p, n, \Sigma, \Xi, \Lambda, \Delta, \Sigma^*,$	$363.03 \pm 0.87$	$538.71 \pm 1.69$	$5043.15 \pm 15.08$	$1.30 \pm 0.03$
	GR	$\Xi^*, \Omega, \Sigma_b, \Lambda_b, \Sigma_b^*, \Omega_b$	$500.11 \pm 3.44$	$624.39 \pm 3.33$	$4923.73 \pm 24.25$	$1.72 \pm 0.07$
2	FB	$p, n, \Sigma, \Xi, \Lambda, \Delta, \Sigma^*,$	$362.94 \pm 0.94$	$538.89 \pm 1.85$	-	$1.30 \pm 0.03$
	GR	$\Xi^*, \Omega$	$499.97 \pm 3.89$	$624.32 \pm 3.78$	-	$1.71 \pm 0.07$

#### 4. Fit of constituent quark masses

For calculating constituent quark masses we used theoretical equations for meson and baryon masses given in Sec. 3. We derived theoretical formulas for FB and GR HFI contribution and then obtained the total theoretical masses of the hadrons we have studied. The corresponding experimental values of the hadron masses are taken from PDG data.<sup>26</sup>

For our calculations we used multidimensional least-square fit of quark masses and hyperfine constant, using subroutine "lfit" from Numerical Recipes in FORTRAN,<sup>27</sup> modified according to the instructions in the last paragraph of Sec. 15.4 of Ref. 27. The equations for hadron masses are linearized by expansion in Taylor series (up to the first order) so we obtained the system of linear equations

for differences between experimental and theoretical hadron masses. The unknown variables in these equations are corrections to the parameters (i.e. corrections to the constituent quark masses and the constant of hyperfine interaction), which are obtained by linear least-square fitting. The uncertainties are estimated during the fitting procedure as square roots of the corresponding diagonal elements of covariance matrix, according to Eq. (15.4.15) of Ref. 27.

The fitting is performed by minimizing  $\chi^2$  between the theoretical and experimental hadron masses, according to the following procedure:

- First, initial values for the constituent quark masses are assumed and used to calculate the initial values of the theoretical masses of hadrons (mesons and baryons), and  $\chi^2$  between theoretical and experimental hadron masses;
- In the next iteration the corrections to the constituent quark masses and the constant of hyperfine interaction are obtained by least-square fitting, and used to obtain new values of the parameters by adding these corrections to the estimates from the previous iteration. The new values of the theoretical hadron masses are then obtained from these corrected parameters, as well as a new value of  $\chi^2$  between theoretical and experimental hadron masses. We repeat this procedure until the fit converges, i.e. while  $\chi^2$  decreases, and finally,
- we choose the set of the constituent quark masses and the constant which gives the least  $\chi^2$ .

Assuming that the number of fitted hadrons (mass equations) is  $N$ , and the number of unknown parameters (constituent quark masses and the constant) is  $m$ , we used the following expression for reduced  $\chi^2$ :

$$\chi^2 = \frac{1}{N - m} \sum_{i=1}^N \frac{y_i^2}{\sigma_i^2}, \quad (17)$$

with  $y_i$  being the  $i$ -th difference between the measured and theoretical hadron masses, and  $\sigma_i$  the  $i$ -th standard deviation:

$$\sigma_i^2 = \sigma_{i,\text{exp}}^2 + \sigma_{i,\text{theor}}^2, \quad (18)$$

where  $\sigma_{i,\text{exp}}$  is an experimental standard deviation given by PDG,<sup>26</sup> and  $\sigma_{i,\text{theor}}$  is a theoretical standard deviation. We added  $\sigma_{i,\text{theor}}$  in quadrature with the experimental errors to avoid having the fit to experiment arbitrarily dominated by the most accurate measurements (Ref. 28, see also Ref. 29). We took  $\sigma_{i,\text{theor}}$  to be proportional to the experimental masses ( $\sigma_{i,\text{theor}} = A \cdot m_{\text{exp}}$ ), and chose the constant of proportionality  $A$  in such a way to yield the reduced  $\chi^2$  as close as possible to 1.

In brief, we fitted four parameters:  $m_u$ ,  $m_s$ ,  $m_b$ ,  $C$ , so that the  $\chi^2$  between measured and theoretical masses is minimized. As we mentioned in Ref. 11, for every system of equations the fast convergence is achieved, even in the case when initial values of parameters differ much from their final values, which tells about goodness of our theoretical model and the fitting method.



In Tables 1 and 2 we give the results from meson and baryon fits, when HFIs are included. The calculation is done by combining the hadrons we chose in Sec. 4. In equations for meson masses there is hyperfine constant labeled by  $C^m$ , and for baryon masses there is hyperfine constant labeled by  $C^b$ . These two constants are not equal, but we can say that they are of the same order of magnitude,  $C^m \sim C^b$ .

Table 3. Absolute differences, in MeV, between experimental masses of mesons and our calculated theoretical masses with FB HFI (third column) and GR HFI (fourth column). Parameter values used here are obtained by  $\chi^2$  fit of all mesons (see fit No. 1 in Table 1 for the third and fourth column).

Meson	$m$ (MeV)	$\Delta m_{\text{FB}}$	$\Delta m_{\text{GR}}$
$\pi^\pm$	$139.57 \pm 0.01$	1.51	3.29
$\pi^0$	$134.98 \pm 0.01$	3.09	1.30
$K^\pm$	$493.67 \pm 0.02$	0.29	55.37
$K^0, \bar{K}^0$	$497.61 \pm 0.03$	3.65	51.44
$\rho^\pm, \rho^0$	$775.26 \pm 0.25$	1.18	39.11
$K^{*\pm}$	$891.66 \pm 0.26$	3.61	0.11
$K^{*0}, \bar{K}^{*0}$	$895.81 \pm 0.19$	0.54	4.04
$\omega$	$782.65 \pm 0.12$	8.57	46.50
$\phi$	$1019.46 \pm 0.02$	18.66	385.06
$B^\pm$	$5279.25 \pm 0.17$	34.04	47.85
$B^0, \bar{B}^0$	$5279.58 \pm 0.17$	34.37	48.18
$B^*$	$5325.20 \pm 0.40$	40.62	58.38
$B_S^0, \bar{B}_S^0$	$5366.77 \pm 0.24$	69.20	95.93
$B_S^{*0}$	$5415.40 \pm 2.20$	45.42	67.53

To compare these two interactions, we calculated the differences between experimental and theoretical masses of the fitted hadrons, and their absolute values presented in Tables 3 and 4. The CPT theorem was not assumed, e.g.  $\pi^+$  and  $\pi^-$  were considered as different points in the meson fits.

## 5. Clustering of hadron groups

Along with the least-square fit method described above, another approach has been applied too. Based on similarity of quark content, the whole set of equations, used in the  $\chi^2$ -fitting method, has been divided into a certain number of subsets chosen in such a way to form a minimal system of equations which could be analytically solved. The Mathematica 9.0 software has been used to solve these systems of equations for mesons and baryons. Typically, there were sets of two equations with two unknown parameters and the sets of three equations with three unknown parameters. For example, from set of Eqs. (3) and (4) a system of equations for  $\pi^+$  and  $\rho^+$  has been formed. This is then analytically solved in  $m_u$  and  $C^m$ . In the case when three equations are chosen to form the system of equations, like for example equations for  $\Sigma^+$ ,  $\Xi^0$  and  $\Sigma^{*+}$  taken from Eqs. (11) and (12), analytically one can get solutions

Table 4. The same as Table 3, but for baryons (see fit No. 1 in Table 2 for the third and fourth column).

Baryon	$m$ (MeV)	$\Delta m_{\text{FB}}$	$\Delta m_{\text{GR}}$
$p$	$938.27 \pm 0.01$	3.28	13.15
$n$	$939.57 \pm 0.01$	1.99	11.86
$\Sigma^+$	$1189.37 \pm 0.07$	7.98	18.07
$\Sigma^0$	$1192.64 \pm 0.03$	11.25	21.34
$\Sigma^-$	$1197.45 \pm 0.03$	16.06	26.15
$\Xi^0$	$1314.86 \pm 0.20$	15.38	5.32
$\Xi^-$	$1321.71 \pm 0.07$	8.53	1.53
$\Lambda$	$1115.68 \pm 0.01$	1.56	10.10
$\Delta$ (mean)	$1232.00 \pm 4.00$	4.61	6.12
$\Sigma^{*+}$	$1382.80 \pm 0.35$	2.58	3.78
$\Sigma^{*0}$	$1383.70 \pm 1.00$	3.48	2.88
$\Sigma^{*-}$	$1387.20 \pm 0.50$	6.98	0.62
$\Xi^{*0}$	$1531.80 \pm 0.32$	2.74	11.85
$\Xi^{*-}$	$1535.00 \pm 0.60$	5.94	8.65
$\Omega$	$1672.45 \pm 0.29$	10.69	24.65
$\Sigma_b^+$	$5811.30 \pm 1.90$	7.09	4.74
$\Sigma_b^-$	$5815.50 \pm 1.80$	11.29	8.94
$\Lambda_b$	$5619.40 \pm 0.60$	2.28	32.79
$\Sigma_b^{*+}$	$5832.10 \pm 1.90$	6.65	18.21
$\Sigma_b^{*-}$	$5835.10 \pm 1.90$	9.65	21.21
$\Omega_b$	$6071.00 \pm 40.0$	76.68	27.94

Table 5. Calculated constituent quark masses  $m_u$  ( $= m_d$ ),  $m_s$ ,  $m_b$  (MeV) and the hyperfine constants  $C^m$  and  $C_\chi^m$  ( $10^7$  MeV<sup>3</sup>) obtained from clustering of equations for meson masses with FB (upper rows) and GR (lower rows) HFI.

System No.	HFI	Combinations of mesons	Quark masses (MeV)			Constant ( $\times 10^7$ MeV <sup>3</sup> )
			$m_u = m_d$	$m_s$	$m_b$	
1	FB	$\pi^+, \rho^+$	308.17	—	—	1.51
	GR		308.17	—	—	2.26
2	FB	$K^+, K^{*+}, \omega$	316.60	475.57	—	1.50
	GR		316.60	475.57	—	2.25
3	FB	$B^+, B^*$	—	—	5001.21	1.80
	GR		—	—	5001.21	2.70
4	FB	$\pi^+, K^+, B_S^0$	—	490.47	4896.04	1.58
	GR		—	490.47	4896.04	2.37

in  $m_u$ ,  $m_s$  and  $C^b$ . In this analysis, four systems of equations for mesons and four for baryons have been formed in the case of FB HFI. The same is done in the case of GR HFI, except for baryons where system of equations formed for  $\Sigma_b^+$ ,  $\Sigma_b^{*+}$  and  $\Omega_b$  did not give real solution. Here, one should also notice that in the case of hadrons which contain  $b$  quark, in order to solve equation it was necessary to include numerical value of  $m_u$  previously obtained from systems of light hadrons. That value is obtained as a mean value of  $m_u$  calculated separately for mesons and for baryons and for both FB and GR HFI.

Table 6. The same as Table 5, but for baryons.

System No.	HFI	Combinations of baryons	Quark masses (MeV)			Constant ( $\times 10^7 \text{ MeV}^3$ )
			$m_u = m_d$	$m_s$	$m_b$	
1	FB	$p, \Delta^{++}$	361.71	—	—	1.28
	GR		508.58	—	—	1.90
2	FB	$\Sigma^+, \Xi^0, \Sigma^{*+}$	375.41	522.62	—	1.27
	GR		495.85	602.10	—	1.47
3	FB	$\Sigma_b^+, \Lambda_b$	—	—	5036.70	1.41
	GR		—	—	4917.23	1.62
4	FB	$\Sigma_b^+, \sigma_b^{*+}, \omega_b$	—	500.15	5039.82	1.29
	GR		—	—	—	—

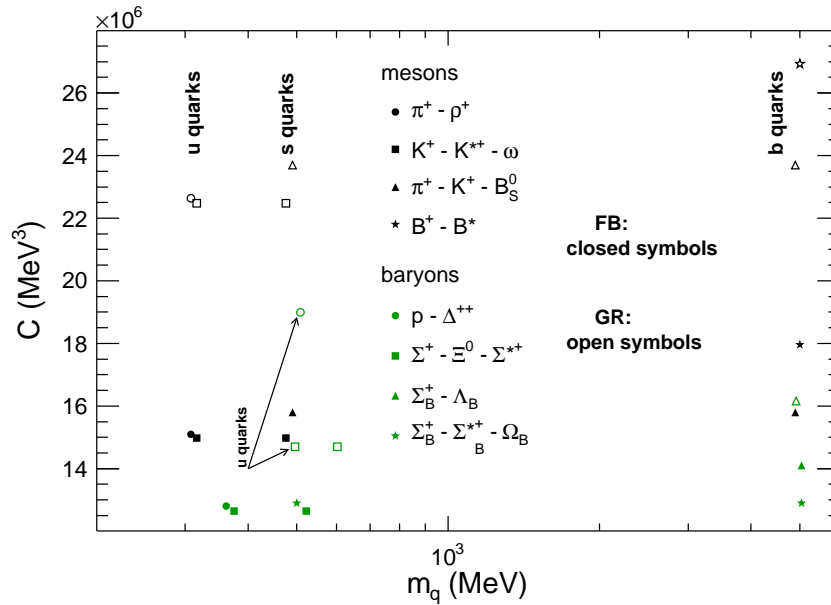


Fig. 1. Masses of constituent quarks confined in different hadrons and the related constants obtained as analytical solutions of the systems of two and three equations formed from sets of Eqs. (3) to (16). The results for mesons (baryons) are depicted with closed black (green) symbols for FB HFI, while in the case of GR HFI they are presented with corresponding open symbols.

As the result, the method described above forms clusters of  $m_q - C$  points as can be seen in Fig. 1. This clustering shows the way how the constituent quark masses and the strength of the interaction constants appear in different environments: mesons - baryons, light hadrons - heavy hadrons within the two analyzed hyperfine interactions. The biggest formed clusters are those characterized by approximately the same quark mass for each of three different quark families ( $u$ ,  $b$  and  $s$ ) but with different constants within the given type of interaction. Here, one

could note that points from both types of interactions belong to the same cluster for a given quark family. The only exceptions seen are green open circle and green open square extracted from baryonic equations for  $p$  and  $\Delta^{++}$  and from equations for  $\Sigma^+$ ,  $\Xi^0$  and  $\Sigma^{*+}$  respectively which gives masses of  $u$ -quarks derived from GR HFI. The corresponding masses are shifted to the position which belongs to the ' $s$ -quarks' cluster. Within these big clusters, one can see smaller clusters which contain points obtained from mesons or from baryons. For the light quarks, typically, mesonic clusters have smaller quark masses and bigger interaction constants with respect to baryonic clusters. Concerning the masses, this observation is clearly seen for the  $u$ -quarks, gets smaller for the  $s$ -quarks and nearly invisible for the  $b$ -quark. On the other hand, the difference between the extracted interaction constant in mesonic and baryonic clusters gets slightly larger going from light to heavy quarks. A similar clustering can also be seen in the case of GR HFI.

The different values of the extracted masses and interaction constants reflect the different environment – mesonic or baryonic – and the type of interaction.

## 6. Discussion

The experimental observation is that there are no free quarks but that they only exist bound in hadrons. This phenomenon is known as confinement. The constituent quark mass is the effective mass of a quark, which is only defined if the quark is confined and bound in the hadron. We calculate constituent quark masses using the improved fitting procedure<sup>28</sup> of groups of mesons and baryons and using method of clustering of some hadron groups.

We give mass formulas for hadrons containing  $b$  quark using FB and GR HFI. We also calculate coupling constants of these interactions and showed that they are not equal but are of the same order of magnitude. We investigate how constituent quark masses, and coupling constants, depend on different hadron environment and how effective these two interactions are.

The obtained results show that quark masses depend on the particular hadron model, and are different for two studied HFIs. On average, FB interaction gives much better fit: the uncertainties of the constituent quark masses are greater by an order of magnitude for GR HFI than for FB HFI in the case of mesons and by a factor of two in the case of baryons. We have to stress that FB interaction is working well for heavy-light mesons and baryons if they contain only one heavy quark, while GR HFI in some cases failed for heavy baryons. GR HFI also fails for  $\phi$  meson.

Just for comparison with the results from least-square fit, we also calculated clustering for some sets of equations and presented it in Tables 5 and 6 and in Fig. 1. Figure shows masses of constituent quarks confined in different hadrons and the related constants:  $x$ -axes represents quark mass (in MeV) and  $y$ -axes the hyperfine constant (in  $\text{MeV}^3$ ). For light mesons (systems (1) and (2) in Table 5), as well as for heavy mesons (systems (3) and (4)) FB and GR give very similar results for

constituent quark masses. For  $m_u$  and  $m_s$  in heavy meson systems we obtained larger values than in light mesons. For baryons, we can notice that in case of light baryons (systems (1) and (2) in Table 6) FB and GR HFI results differ which is opposite from case of light mesons: GR interaction gives larger values for  $m_u$  and  $m_s$  than FB. When comparing heavy and light baryons (systems (3) and (4)) we have the greater value for  $C^b$  in heavy baryons than in light baryons. In Table 6, we do not have values for GR HFI for the fourth system, because physically realistic values could not be obtained when solving the equations. We can conclude that FB HFI is more accurate interaction than GR HFI.

According to the clustering procedure, both interactions FB and GR have a similar behavior, but in the case of heavy baryons FB HFI is better because GR HFI did not give good results in some cases (i.e. system (4) in Table 6). From this method, we can conclude that constituent quark masses are very sensitive to environment of different hadrons, as well as the values of constants. Values of the constants are somewhat higher in hadrons which contain  $b$ -quarks.

In Refs. 30, 31 it was shown that in the constituent quark model, the Feynman-Hellmann theorem and semi-empirical mass formulas can be applied to give useful information about the masses of mesons and baryons. We obtained that the Feynman-Hellmann theorem is working well in case of mesons and baryons with FB HFI, as well as with GR HFI.

## 7. Conclusions

In order to determine the constituent quark masses we have used two methods: (1) least-square fits of both light and open-bottom heavy light hadrons, (2) clustering of hadron groups. We improved fitting procedure used in method (1).<sup>11,28</sup>

In the previous work, least-square fit gave similar results for FB and GR HFI for light mesons.<sup>11</sup> For heavy light hadrons studied in this paper, we find that FB HFI gives much better fits. This could have been expected given the  $SU(3)_F$  nature of the GR HFI. More appropriate spin-flavor HFI for light heavy hadrons will be the subject of further investigation.

The FB HFI gives reasonably good fits for all hadrons that we have considered – including the open bottom ones, especially when one takes into account the simplicity of the model. Further improvements are likely to be achievable only in much more elaborate, multiparameter models.

We have confirmed that constituent quark mass depends on the type of the hadron where quarks are confined and on the particular hadron model. We show that, in general, quark mass has a larger value in baryons than in mesons. Also, it depends on particular type of mesons and baryons, i.e. it is not the same in different mesons, but it is more similar than in baryons.

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